INDEX NUMBERS

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B. S., Kansas State University, 1963

A MASTER'S REPORT

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Statistics

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1966

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I. INTRODUCTION

Introductory Statements

In order to facilitate the peaceful coexistence of sectors of an economy, the government needs an adequate measure of each sector's contribution to the nation's wealth so that it can adopt policies which will ensure each sector an equitable share in the increase in wealth. In an economic system where factors of production compete freely, the price of a factor of production is a measure of its value under normal conditions. However, certain phenomena, such as business cycles, disturb the stability of prices as measures of value and incite an uneven distribution of national income. During a business downswing, the primary producing sector, composed mainly of farmers, suffers a greater loss of income than the secondary producing sector composed mainly of manufacturers, because agricultural products, in contrast to manufactured products, cannot be stored for a longtime without incurring a loss. In such situations, the government is obligated to formulate policies which will help compensate farmers for their losses. In order to compare prices at the unfavorable period to those of a normal period it is convenient to use a measure which is independent of the monetary unit employed in the exchange of commodities. This measure is called an index number.

Index numbers are numbers which are used to indicate the average change in a group of related variables (Davis, 1941). They may be applied to values in an economic time series. For

example, when it is desired to measure the change in the level of the prices of a certain group of commodities between two time periods, the prices of representative commodities at the two periods are compared. A simple price index number is the ratio of the weighted average of the prices of commodities. Index numbers may also be computed for data representing a number of time series. When data relating to variables of interest, have been collected, it is possible to calculate an index number for each of the variables and later combine these indices into a composite index. The series may relate to prices, production, volume of trade or any other factor subject to temporal or spatial variation.

An index number is a special type of index which is expressed as a ratio, (an index ratio) (Florence, 1929). Index numbers differ from other kinds of indices in the characteristics of the variables compared. In order to obtain an index number, a pair of values of the same variable at two periods of time are used. Other type of indices, such as mortality rates, give the ratio of different variables, for example, the number of deaths for one thousand people. Still other types of indices like the index of efficiency of workers in a department of a company use as their denominator, an average, or some other standard value.

Construction of Index Numbers

After a clear statement of the purpose for which the index

number will serve has been made, the investigator should consider the following important points.

1. Sampling: The precision of an index number depends partly on the choice of the number and type of items to be included in the sample, the time of sampling, the reporting establishment, the method of collection of data, and the units of measurement. Therefore, careful consideration should be given to this phase in the construction of the index.

2. The weights: In general, the relative importance, or weight, of each item in a population is different. When thought is not given to this aspect of the construction of an index, an equal weight for the items is implied, and the index is said to be an unweighted index. An unweighted index is usually biased. The weight may be the quantity of the commodity produced or consumed at the base period, as in the case of a price index, or it may be the value (volume of production times representative price) of the commodity at the base period, as in the case of the volume index.

3. Comparability of samples: In order for the comparison of variables in the two periods to be valid, it is necessary that adequate data are available for the two periods, so that every item in the later sample has a counterpart in the earlier one. If an article whose price has been included in the sample at an earlier time becomes obsolete, the price of a substitute should be taken and proper adjustment made for the difference in the item before any comparison of the two periods is made. Often,

an allowance has to be made in recording the price of a commodity whose quality has improved over time.

4. Choice of the base period: The choice of the first period, usually called the base period, is important since it will serve as a standard of comparison. It is usually regarded as a "normal" period where the conditions of the variables are unique, it usually covers several years but it is not uncommon for it to cover just one year or even one month. The value of the variables in the base period is taken as 100, and the value of the second period, sometimes called the current period is expressed as a ratio or a percentage of the base period.

5. Choice of formula: Two formulas may be used for computing an index number, depending on the type of data and the form of the weights available: (i) The average of relative formula, or (ii) The aggregative formula.

Uses of Index Numbers

In general, all index numbers can be grouped in two main categories: Price Index Numbers and Volume Index Numbers (Davies, 1922).

Price Indices

Wholesale Price Index. A common practice of grouping, for the purpose of computing an index number, is on the basis of I. Raw materials, and II. Manufactured goods. Their subdivisions are as follows:

I. Raw materials
   a. Agricultural products
   b. Animal products
   c. Forest products
   d. Mineral products

II. Manufactured goods
   a. Producers' goods
   b. Consumers' goods

Items are generally grouped according to the similarity of the patterns of price movements. Within the raw material group, prices of the agricultural products are affected more than any other subgroup, by weather conditions. Prices of mineral products are influenced most by forces affecting business cycles. When the appropriate method of averaging and weighting have been chosen for each subgroup, a separate index may be computed for each of the subgroups, and these indices may be combined with their appropriate weights (usually expenditures) to obtain an overall index. The United States Bureau of Labor Statistics collects data on wholesale prices and computes a Wholesale Price Index which indicates changes in the general price level. This is a comprehensive index which is useful to all industries. However, other types of wholesale price indices which will serve specific purposes may be computed.

For the monthly Retail Price Index computed by the United States Bureau of Labor Statistics, the total money prices of each article are taken and the average is obtained by dividing this sum by the number of dealers reporting the prices of the article. This gives the representative price. Each price is then multiplied (weighted) by the quantity of the commodity consumed. The resulting products are then summed and compared with those of the previous month. The lack of standardization in retail goods,

variations in business practices among regions, and local customs contribute to the imprecision of this index.

Index Numbers of Price and Buying Power of Farm Products. The United States Department of Agriculture compiles (a) an index number designed to measure changes in prices received at the farm for chief agricultural products and (b) an index number for measuring changes in farm prices of livestock products. These two indices are combined to measure changes in the average price of farm products. A third index based on the Bureau of Labor Statistics wholesale price index is computed by using the Bureau's index but excluding farm products and foods. Then an index intended to measure the purchasing power of farm products is expressed as a percentage of the index of change in average price of farm products to the corrected index of change in wholesale prices. This index serves as a basis for the government's price support of farm products.

Index Number of Money Wages and Real Wages. The problems involved in computing this index are in averaging the costs of labor. Should the average be based on the hourly rate of pay or weekly earnings which are of two types; (a) full-time earning which is based on forty hours and (b) the actual earning which includes overtime payments also.

Four types of indices can be obtained from wage statistics.

1. An index of wage-rates measures the unit price of labor during the legal number of weekly hours.

2. An index of the average hourly earnings reflects the average return to labor per unit of time worked. This is affected by the basic wage rates, the average amount of additional payment for overtime, the shift differentials, and other supplementary payments.
3. An index of average weekly earnings combines the effects of the changes in the actual length of work week with those of fluctuations in the average weekly earnings.
4. An index of annual earnings reflects the degree of employment during the year as well as the level of average weekly earnings.

Wage rates are used for comparative interindustry or interregional differentials in the basic rate for classifying, summarizing and providing a quantitative record of individual transactions which are within the sphere of economic activity.

The Consumer Price Index. This index indicates the changes in prices of the commodities which are usually purchased by clerical workers and urban wage earners. Prior to 1945, the index was called an index of the Cost of Living. The Federal government considers this index when it formulates monetary and fiscal policies and uses the index in the revision of welfare support programs. More will be said about this index later in the report.

Volume Indices

Aggregative Index of Production. The same classification which has been used under the Wholesale Price Index may be used. The value aggregates, which are the commodities produced times their prices, can be obtained for each period and these aggregates may then be expressed as relatives to obtain the required index. Here, prices serve as the weights, but the individual commodity may also be weighted by a weight which is proportional to the number of the manufacturing operations required in the fabricating process.

Index of Cyclical Change in Physical Volume of Production. Business cycles are often the object of primary interest in studying the physical volume of production. When annual data are used in constructing the index of cyclical changes, it is necessary to account for the secular trend. A trend line may be fitted to each of the constituent series by the least squares method, for example. The actual items are then expressed as percentages of the corresponding trend value, and the required index for a given year is obtained by taking a weighted arithmetic average of these percentages for the year. The resulting adjusted index is in terms of relatives, but the relatives refer to a hypothetical "normal" instead of any fixed base. A normal value is the same as a trend value.

When the construction of an index number is based on monthly data, it may be necessary to adjust for seasonal variation.

Deflation as a Step in the Analysis. Since many series of

economic data are expressed in monetary units, they are subject to distortion due to changes in the price level. Therefore, it is necessary to correct such series for changes in the price level. For example, the index of wage rates may not give a true picture of the wellbeing of wage earners. It is necessary to convert the index of money wages to the index of real wages, which indicates changes in the cost of living. That is,

$$\text{Index of Real Wage} = \frac{\text{Index of Relative Earning}}{\text{Consumer Price Index}}$$

Sometimes, the rise in prices is partly due to quality changes in commodities. Therefore, the price index should be deflated by an appropriate quality index. In certain instances, people incorrectly equate the value of an index which is easy to compute, to the value of a related index which is not so easy to compute. It is incorrect to equate the index of the volume of construction to the index of the value of building contracts, for example. The value of building contracts awarded in any one year depends not only upon the actual volume of construction but also upon the costs of building materials and building labor. Therefore, the index of the volume of construction is the ratio of the index of the value of building contracts to the index of the building costs.

This process of correcting an index for the influence of exogenous variables is called deflation.

Organization of Report

In this report, a discussion of the common index number formulae together with their relative precisions will be followed by the development of a Price Index Theory to approximate a Cost of Living Index. Then there will be a discussion of the relationship between the True Cost of Living Index and the Consumer Price Index which is used to approximate it. Finally some of the differences between a price index and a quality index will be mentioned.

II. FORMAL PROPERTIES OF INDEX NUMBERS

In this section, the relationships of the common formulae which are used in the computation of an index number and their precisions will be discussed. Three errors which are committed in the computation of the index will also be discussed.

Common Formulae and Their Precisions

The importance of weights in the construction of an index number has been indicated in the introduction. However, there is a choice between two sets of weights. In order to compute an index of price change one may choose either the set of quantities of commodities consumed in the base period or the set of quantities of commodities consumed in the current period as measures of the importance of individual prices (Marris, 1958). Hence the formulae most often used to compute index numbers are:

Ia. Aggregative formula
$$\frac{\sum_{i=1}^{n} q_{io}p_{i1}}{\sum_{i=1}^{n} q_{io}p_{io}} \qquad (2.1)$$

Ib. Average of Relatives Formula
$$\frac{\sum_{i=1}^{n} q_{io}p_{io}\left(\frac{p_{i1}}{p_{io}}\right)}{\sum_{i=1}^{n} q_{io}p_{io}}$$

where p_i represents the price and q_i represents the quantity of the i^{th} commodity. The subscripts, o and 1 indicate the periods of reference. These base-weighted formulae are credited to Laspeyres. Formula Ia indicates the effect of price change from

period-o to period-1, using the period-o quantities as weights. Formula Ib indicates the same effect, using the period-o expenditures as weights.

IIa. Aggregative formula
$$\frac{\sum_{i=1}^{n} q_{i1}p_{i1}}{\sum_{i=1}^{n} q_{i1}p_{io}} \qquad (2.2)$$

IIb. Average of Relatives Formula
$$\frac{\sum_{i=1}^{n} q_{i1}p_{io}\left(\frac{p_{i1}}{p_{io}}\right)}{\sum_{i=1}^{n} q_{i1}p_{io}}$$

where IIa measures the effect of price change on the value of money, using period-1 quantities as weights; and IIb uses the expenditures on period-1 collection at period-o prices as weights. Actually the second type of formula gives the inverse of the index of the change from the second period to the first. These are known as Paasche's formula. $\sum_{i=1}^{n} q_{io}p_{io}$ denotes the total expenditures in period-o and $\sum_{i=1}^{n} q_{i1}p_{i1}$ denotes the total expenditures in period-1. $\sum_{i=1}^{n} q_{io}p_{i1}$ is the amount of money required to buy period-o quantities at period-1 prices, and similarly, $\sum_{i=1}^{n} q_{i1}p_{io}$ is the amount of money required to buy period-1 quantities at period-o prices. These two formulae (I and II) which are used to measure the same change that has occurred between two time periods often give different values.

Formula Error

Irving Fisher measured formula error by two tests (Davis, 1935): (a) The time reversal test and (b) The factor reversal test (sometimes called the product test).

(a) The time reversal test. In order that a formula will satisfy this test, the product of an index P_{o1}, where period-o is used as the base, by the index P_{1o}, where period-1 is used as the base should equal unity. That is,

$$P_{o1} \cdot P_{1o} = 1$$

otherwise the formula is inadequate and the measure of the percentage of the joint error, E_1 of the two indices is

$$E_1 = P_{o1} \cdot P_{1o} - 1 \qquad (2.3)$$

If $P_{o1} \cdot P_{1o} > 1$, then the formula is biased upwards, as is the case with Laspeyres' formula. When Paasche's formula is used, $P_{o1} \cdot P_{1o} < 1$, and it is, therefore, biased downwards.

(b) Factor reversal test. The total value of production for a given year is the sum of the products of the volume of various commodities by their average prices. If the total value of production, as measured by an adequate index, shows an increase over the preceding year, the increase must be due to a growing volume of production, or rising prices or both. Therefore, when the same set of data is used to compute the three types of indices, the index of a price change, P_{o1} multiplied by the index of physical production, Q_{o1} must equal the index of value of production, V_{o1}.

When the equality does not hold, the percentage measure of the joint error E_2 of the quantity index and the price index (Mudgett, 1951) is:

$$E_2 = \frac{P_{o1}\ Q_{o1}}{V_{o1}} - 1 \qquad (2.4)$$

121 of 134 index formulae tested by Irving Fisher did not pass these tests. Both Laspeyres' formula and Paasche's formula give good estimates of the index. More precise estimates may be obtained when either of the following formulae is used.

1. Cross of Laspeyres' and Paasche's formulae:

a. Arithmetic Cross: $$\frac{1}{2}(L + P) = \frac{1}{2}\left[\frac{\sum q_{io}p_{i1}}{\sum q_{io}p_{io}} + \frac{\sum q_{i1}p_{i1}}{\sum q_{i1}p_{io}}\right] \qquad (2.5)$$

b. Geometric Cross or "Ideal" formula: $$\sqrt{L \cdot P} = \sqrt{\frac{\sum q_{io}p_{i1}}{\sum q_{io}p_{io}} \cdot \frac{q_{i1}p_{i1}}{q_{i1}p_{io}}} \qquad (2.6)$$

2. Cross-weight Aggregative formulae:

a. Arithmetic Cross-weight Aggregative: $$\frac{\sum (q_{io} + q_{i1})p_{i1}}{\sum (q_{io} + q_{i1})p_{io}} \qquad (2.7)$$

b. Geometric: $$\frac{\sum \sqrt{q_{io}q_{i1}}\ p_{i1}}{\sum \sqrt{q_{io}q_{i1}}\ p_{io}} \tag{2.8}$$

3. Fixed-weight Aggregative: $$\frac{\sum q_{ia}p_{i1}}{\sum q_{ia}p_{io}} \tag{2.9}$$

where q_{ia} are quantities that in some way measure the relative importance of the prices. L represents Laspeyres' formula and P represents Paasche's formula.

Formula 1b. was called "ideal" by Fisher because it met the requirement of the two tests. It is seldom used because considerable labor is required for its computation. The last formula was judged as a fair formula, but many economists disfavor it because it uses weights which are often unrelated to the data. The others were judged by Fisher as being superlative. The individual formulae are more suited for different forms of data.

Tests

Some other criteria (Frisch, 1930) for choosing a formula are:

a. Identity test which requires $P_{oo} = 1$.

b. Circular test which requires that

$$P_{12} \cdot P_{23} = P_{13}.$$

c. Commensurability tests which requires that the index be independent of the units of measurement. If for any commodity, p is replaced by λ p and q is replaced by q/λ both at time o and 1, then P_{o1} shall remain unchanged.

d. Determinateness test which requires that P_{01} does not become zero, infinite or indeterminate if an individual price or quantity becomes zero.

e. Proportionality test which requires that when individual prices change in the same proportion from period-o to period-1, the index P_{01} equals the common factor of proportionality.

Using the calculus and the assumption that the index number formula was a continuous function of prices and quantities, Frisch explored the forms of index numbers that would satisfy one or more tests simultaneously. He concluded that Laspeyres' and Paasche's formulae would satisfy only tests (c), (d), and (e), and that (c), (d), and (e) could not be satisfied at the same time.

Only three of the above formulae have survived:[1] (1) Laspeyres' index, (2) Paasche's index, and (3) Arithmetic crossed-weight aggregative. Laspeyres' formula is favored over the other two because (a) it requires only minimal data and (b) it is the easiest to interpret. In order to compute Laspeyres' index, quantity data are only required for the base period, so that in the subsequent periods only price data need be collected.

The Arithmetic crossed-weight aggregative index is an approximate index and may be used to estimate percentage change between two adjacent periods, only. If the desire is to compute a series of index numbers then the geometric cross-weight aggregative is the appropriate formula.

When the periods of comparison are close and the population's

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[1] The Price Statistic of the Federal Government. National Bureau of Economic Research, Inc., 1961.

taste remains constant, then Laspeyres' and Paasche's indexes may give the same value (Mudgett, 1951).

Chain vs. Fixed Base Indices

The precision with which an index indicates the magnitude of the change depends as much on the intervening period as on the lengths of the periods of comparison. An examination of the intervening period may throw light on the course of the change. Therefore, it is worthwhile to observe the changes during short time periods in order to make a suitable inference for any long period, i.e., to use a series of index numbers.

There are two types of series of index numbers depending on the selection of the base period: (1) The Fixed Base Index and (2) The Chain Index. The first type usually uses the period one as the base, whereas the base is shiftable in the second type of series. Thus the second type of series gives a better opportunity for observing the form of the changes over a long period. However, for a comparison of two periods far apart neither value of the two series can be taken as a measure of a change because a long period is usually nonhomogeneous with respect to the variables under consideration.

Suppose that first year prices are to be compared with fourth year prices. The Fixed Base Aggregative method gives

$$P_{14} = \frac{\sum q_{11}p_{14}}{\sum q_{11}p_{11}} \tag{2.10}$$

while the Chain method gives

$$P_{14} = \frac{\sum q_{11}p_{12}}{\sum q_{11}p_{11}} \cdot \frac{\sum q_{12}p_{13}}{\sum q_{12}p_{12}} \cdot \frac{\sum q_{13}p_{14}}{\sum q_{13}p_{13}} \qquad (2.11)$$

by Laspeyres' formula.

Thus far, only formula errors have been discussed. Other significant errors are: (1) Sampling errors and (2) Homogeneity errors.

Sampling Errors

Sampling errors can arise through the inability of the investigator to collect a truly representative sample. The population is usually finite, and caution needs to be exercised in order to select an optimum sample. This error can be minimized by proper instruction of the agents who collect the data. Some other sources of error are incorrect stratification of the sample and improper allocation of weights to items in the sample. An item is usually selected to represent a group and this item is priced every time a sample is taken. This procedure excludes the chance of another item's selection and is therefore an inefficient way to sample. Banerjee and Adelman have suggested ways to improve the construction of index numbers.

Adelman (1958) compared two possible methods of construction with the familiar method. One of the methods, together with the familiar one, will be given here. It is assumed the weights of items within each stratum is one, and the relative weight of each

stratum has been determined.

In the first method the weighted means of a randomly selected equiprobable sample is used in each stratum:

$$\hat{R}_1 = \frac{\sum_{j=1}^{n} R_{ij} W_{ij}}{\sum_{j=1}^{n} W_{ij}} \tag{2.12}$$

where R_{ij} denotes the price relative of the j^{th} item in the i^{th} stratum, and W_{ij} denotes the corresponding weight. There are n items in a stratum, and $\sum_{j=1}^{n} W_{ij} = 1$, $\hat{R}_1$ is a ratio of two random variables (numerator and denominator are variables) which are not independent and so the expected value, E, of this ratio is not the ratio of the expected values.

For if $\sum_{j=1}^{n} R_{ij} W_{ij} = x$ and $\sum_{j=1}^{n} W_{ij} = y$

then $$E(\frac{x}{y}) = \frac{Ex}{Ey} (1 - \rho_{xy} V_x V_y + V_y^2) \tag{2.13}$$

where V_x is the coefficient of variation of x, V_y is the coefficient of variation of y, ρ is the correlation coefficient between x and y. The last two terms in the parenthesis constitute a bias. Hence $\hat{R}_1$ is a biased estimate.

In the second method one assigns to each of the n_i items a probability proportional to its weight, W_{ij}, draws a sample with replacement, and then takes a simple arithmetic average:

$$\hat{R}_2 = \frac{1}{n} \sum_{j=1}^{n_i} R_{ij} \tag{2.14}$$

The contribution of each R_{ij} to the expected value of $\hat{R}_2$ is $W_{ij}\ R_{ij}$ and the estimate is unbiased, for

$$E(\hat{R}_2) = \frac{1}{n_i} \sum_{j=1}^{n_i} E(R_{ij}) = E(R_{ij}) \qquad (2.15)$$

since R_{ij} is the same on all drawings. The probability that the j^{th} item will be chosen on the i^{th} drawing is W_{ij} so that

$$E(\hat{R}_2) = E(R_{ij}) = \sum_{j=1}^{N_i} W_{ij}\ R_{ij} = R_i \qquad (2.16)$$

Precision: Adelman made some empirical studies using the three methods she analyzed and found that R_2 (which is Adelman's R_3) had the smallest variance. The variance of $\hat{R}_2$ is given by

$$\sigma^2_{\hat{R}_2} = \frac{N_i}{n_i} \sum_{j=1}^{N_i} (W_{ij}\ R_{ij} - \frac{R_i}{N})^2 = \frac{N_i^2}{n_i}\ \sigma^2_{W_iR_i} \qquad (2.17)$$

Since the estimate W_iR_i is unbiased, an index number, which is a weighted mean of averages of individual strata, based on $\hat{R}_2$ will be unbiased. Therefore, using the second procedure, and a sample of price relatives drawn with replacement from stratum i, with probability proportional to W_{ij} , $\hat{R}_i = \sum^{n_i} R_{ij}/n_i$, $E(\hat{R}_i) = R_i$ and $\sigma^2_{\hat{R}_i} = \frac{\sigma^2_i}{n_i}$. If individual strata indexes are

combined with strata weights,[2] then

$$\hat{R} = \frac{\sum_{i,j} \left(\sum W_{ij} \right) \hat{R}_i}{\sum_i \sum_j W_{ij}}$$

and

$$\sigma^2_{\hat{R}} = \sum \left(\frac{\sum_i W_{ij}}{\sum_i \sum_j W_{ij}} \right)^2 \frac{\sigma^2_i}{n_i}$$

where R_{ij} is the j^{th} price relative within the i^{th} stratum, $\hat{R}_i$ is the estimate of the average price relative of stratum i, and σ^2_i is the variance of the price relatives in the i^{th} stratum.

Adelman stated that this method might be a little more costly than the usual judgement sampling but it will permit the use of standard inference techniques to test the significance of the difference between two index numbers. An additional list of products will need to be drawn and priced every time a previous sample is priced for the second time but the procedure affords an optimum frequency of sample revision. One can easily make adjustment for quality of new items, taking into account appropriate weights and the probabilities of selection in the next drawing.

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[2]The Price Statistics of the Federal Government.

Homogeneity Error

For two time periods to be completely comparable, the same number of commodities should be available in the two situations. Generally, this criterion does not hold for most periods compared. Suitable matches can not always be made in the two samples because technology changes constantly. The qualities of commodities change, and the production of other commodities is discontinued. In order to decrease this error, comparisons are made among periods which are close together and procedures like splicing are used to adjust for quality change in the index. Stone (1956) suggested subdividing a commodity into as many separate products as necessary to reflect adequately the existing differentials and treat each subdivision as a separate commodity. This approach, however, will call for a much larger number of commodities and a much longer time to compute the index.

III. A DEVELOPMENT OF A PRICE INDEX THEORY

Economists grapple with the problem of increasing the economic welfare of the people. They believe that the level of welfare will increase when more goods are made available to the people for consumption. It has been shown that a rising level of prices may cause a deception in the rising standard of living. Therefore, an index which indicates the change in the standard of living may be deflated for the change in the price level. In order to measure the effect of the average change in prices on the consumption of goods by a group of people, a cost of living index is used.

The True Cost of Living Index is the ratio of an average family's expenditures, under two different price systems. It is assumed that the taste of the family remains constant at the two periods and its income is just sufficient to buy sets of quantities of commodities which give the same amount of satisfaction. It is recalled that none of the formulae given in last section satisfy all of the tests for uniqueness. The failure to determine a unique formula by the test approach has led economists to refer the problem of comparing the levels of prices at two time periods to that of comparing utilities (the satisfactions derived from using goods). By the comparison of the utilities of different sets of commodities which an average individual consumes, A. A. Konus (1939) showed in his article, that if, at the present time, an individual is guaranteed the money to gather the same collection of commodities which he consumed at an

earlier time period but he chooses a different combination of commodities, then he is enjoying a higher level of living standard. He assumed that the taste of the individual remains constant during the two periods. Two situations can differ with respect to factors such as the taste and consumption habits of individuals, and the choice of commodities available for consumption.

Economists analyze such situations with the assumption that only one factor varies while the others remain constant. This is the procedure used in the Price Index Theory to determine the True Cost of Living Index. A preliminary to the understanding of the analysis is the understanding of an indifference surface, which is facilitated by the Theory of Revealed Preference.

Suppose that in a given market there are n commodities. Let $[q_o]$ represent a set of quantities $q_1^o, q_2^o, q_3^o, \ldots, q_n^o$; $[q_1]$ represents another set $q_1^1, q_2^1, q_3^1, \ldots, q_n^1$; . . . $[q_m]$ represents $q_1^m, q_2^m, q_3^m, \ldots q_n^m$; . . . . Various sets of quantities can be formed from the n commodities in this manner.

Axiom 1. If $[q_o]$ is preferred to $[q_1]$, then $[q_1]$ must never be revealed to be preferred to $[q_o]$.

Axiom 2. If $[q_o]$ is revealed to be preferred to $[q_1]$ which is revealed to be preferred to $[q_2]$ which is revealed to be preferred to $[q_3]$, then $[q_3]$ must never be revealed to be preferred to $[q_o]$.

This axiom ensures the transitivity of revealed preference

(Henderson and Quandt, 1958).

When a consumer conforms to these two axioms, he is said to possess a preference map. This idea will now be tied to that of utility.

There can be situations where the consumer is indifferent to the choice of either of two sets of commodities. This statement implies that the two sets are equally useful to him. However, the preference of one set of quantities over another signifies that one set is more useful than the other set. Thus the usefulness (utility) of a given collection of goods is a function of the quantities of the goods. But the difference between the two levels of utility may not be measured quantitatively. In order to differentiate between the two levels, one can label them. Therefore, in analyzing situations like this , the sets of quantities which give the consumer equal satisfaction are represented by a curve, in two dimensional space, called an indifference curve. The utility level of these sets is assigned a number. Another collection of sets which give higher satisfaction are represented by an indifference curve which lies to the right of the earlier curve; the level of utility of this curve is assigned a larger constant. However, the two constants are not measures of magnitudes of the two levels of utility.

From the foregoing discussion, it can be inferred that in a given market, a consumer with a limited amount of income chooses quantities of commodities in such a way as to maximize his satisfaction. Assuming there are only two commodities, his

budget constraint can be written as

$$R = p_1x_1 + p_2x_2 ,$$

where R is the expenditure, and p_1 and p_2 are the prices of commodities X_1 and X_2, respectively.

Assumptions:

1. Quantities $x_1, x_2, . . . x_n$ are capable of continuous variations.
2. The function U $(x_1, x_2, . . . , x_n)$, is continuous and it possesses continuous first and second order derivatives.

The consumer seeks the quantities x_1 and x_2 which will make the utility function $U = U(x_1, x_2)$ a maximum. It can be shown (Henderson and Quandt, 1958) that at the maximum point of utility

$$U_1 = \frac{dU}{dx_1} = \frac{dU}{dx_2} = U_2 = 0$$

where U_α is the derivative of the function U with respect to x_α . U_α is called the marginal utility of X_α . The marginal utility of any commodity X_α is the utility derived from the last dollar spent on the commodity. Further, at the maximum point of utility, the marginal utility divided by price must be the same for all commodities. The problem is (1) to determine an amount of income which will give this consumer the same satisfaction in another situation with a different set of prices, and (2) to calculate in two situations where equivalent money incomes are known, and which differ only in respect to prices, the true

measure of the variation of the cost of living (Staehle, 1935). The consumer's equilibrium is defined by

$$\frac{1}{p_{1o}} \cdot \frac{dU}{dx_{1o}} = \frac{1}{p_{2o}} \cdot \frac{dU}{dx_{2o}} \qquad (3.1)$$

which is a condition of maximizing satisfaction, subject to

$$x_{1o}\, p_{1o} + x_{2o}\, p_{2o} = R_o \qquad (3.2)$$

where R_o is the individual's total expenditure at period-o. U is the utility, p_{1o} is the price of commodity X_1, p_{2o} is the price of commodity X_2 at the period-o. Equation (3.2) can be rewritten as

$$x_{1o} = \frac{R_o}{p_{1o}} - \frac{p_{2o}}{p_{1o}}\, x_{2o} \qquad (3.3)$$

which is an equation of a straight line with slope p_{2o} / p_{1o} and the intercepts R_o / p_{1o} and R_o / p_{2o} on the ordinate and abscissa respectively (See Figure 1). The particular expenditure line is represented by A_oB_o. The maximizing situation, as expressed by equation (3.1), involves the determination of a point on the expenditure line A_oB_o which lies on the highest indifference curve; given that the indifference curves are convex toward the origin. This criterion is satisfied by assumption 2 above. This is point Q_o, where A_oB_o is tangent to an indifference curve and whose coordinates are therefore q_{1o}, q_{2o}, the equilibrium values of the quantities purchased. Assuming that prices change to p_{11} and p_{21}, what money income R will secure the same satisfaction as that obtained from R_o with the old prices. With p_{11}, p_{21}, there will be a series of parallel expenditure lines

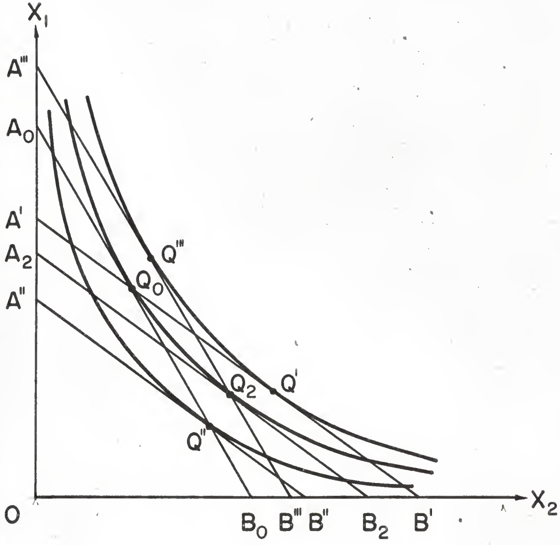

FIG. I. INDIFFERENCE CURVES AND INDEX NUMBERS.

determined by

$$q_{11} = \frac{R}{p_{11}} - \frac{p21}{p_{11}} \; q_{21} \qquad (3.4)$$

with slope p_{21} / p_{11} and situated more or less distant from o according as greater or smaller values are given to R. Some of these expenditure lines are represented by A_2B_2, A"B". One such line, say $A_\alpha B_\alpha$ will touch the same indifference curve as A_oB_o. The particular value of R corresponding to the straight line $A_\alpha B_\alpha$ (call it R_α) is the income sought, and $I = \frac{R_\alpha}{R_o}$ is the true index of change in cost of living. From the figure, it is seen that A'B' passes through Q_o and therefore, corresponds to particular expenditure R' which can buy the old quantities q_{1o}, q_{2o} at the new prices p_{11}, p_{21}. Then

$$R' = p_{11}q_{1o} + p_{21}q_{2o} \qquad (3.5)$$

But an individual with R' in the second situation would rather move along the expenditure line A'B' to Q', the point of tangency with a higher indifference curve than that touched by A_oB_o. Given that the slope of the indifference curve is convex towards the origin, it will be seen that $R' > R_2$ which yields with new prices exactly the same satisfaction as R_o under old prices (i.e., A'B' is further from the origin than the parallel line A_2B_2 which is tangent to the same indifference curve as A_oB_o). Thus

$$R_\alpha < R' = p_{11}q_{1o} + p_{21}q_{2o}$$

and so

$$\frac{R_\alpha}{R_o} < \frac{R'}{R_o} = \frac{p_{11}q_{1o} + p_{21}q_{2o}}{p_{1o}q_{1o} + p_{2o}q_{2o}}$$

This reasoning is valid for any number of articles, so that in general since $I = R_\alpha / R_o$,

$$I = \frac{R_\alpha}{R_o} < \frac{\sum_{i=1}^{n} p_{i1}q_{io}}{\sum_{i=1}^{n} p_{io}q_{io}} = P_o \qquad (3.6)$$

where P_o is Laspeyres' index comparing the price system in period-1 with that of period-o; and it is thus seen to be larger than the "true" index, which is the change in money income required by an individual, with a given money income, to retain the same level of satisfaction under a changed price system as he had in the basic price situation.

Starting from the second situation, by substituting in equations (3.1) and (3.2) p_{11} and p_{21} for p_{1o} and p_{2o},

$$\frac{1}{p_{11}} \cdot \frac{dU}{dq_{11}} = \frac{1}{p_{21}} \cdot \frac{dU}{dq_{21}} \qquad (3.7)$$

and $q_{11}p_{11} + q_{21}p_{21} = R_2$, getting the equivalent values of q's. In the diagram, these are coordinates of Q_2, the point of tangency with an indifference curve of expenditure line A_2B_2 whose equation now is

$$q_{11} = \frac{R_2}{p_{11}} - \frac{p_{21}}{p_{11}} q_{21} \qquad (3.8)$$

The cost of buying the new quantities q_{11} and q_{21} under the

original price system p_{1o} and p_{2o}, will be $q_{11}p_{1o} + q_{21}p_{2o}$, which may be called R'''. (A''' B''') is the straight line representing the expenditure line that corresponds to the money income R''' under the first price system p_{1o}, p_{2o}. The point of maximum satisfaction on this line is Q''' which lies on a higher indifference curve than Q_2. The expenditure line which under the first price system would yield satisfaction equal to that of Q_2 under the second price system is clearly A_oB_o, which touches (at Q_o) the same indifference curve as that on which Q_2 lies and which involves an expenditure R_o. But owing to the downward convexity of indifference curves, R''' lies further from the origin than R_o, thus $R''' > R_o$, i.e., $q_{11}p_{1o} + q_{21}p_{2o} > q_{1o}p_{1o} + q_{2o}p_{2o}$ therefore

$$\frac{q_{11}p_{1o} + q_{21}p_{2o}}{q_{11}p_{11} + q_{21}p_{21}} > \frac{q_{1o}p_{1o} + q_{2o}p_{2o}}{q_{11}p_{11} + q_{21}p_{21}}$$

reversing and generalizing

$$I = \frac{R_2}{R_o} > \frac{\sum q_{i1}p_{i1}}{\sum q_{i1}p_{io}} = P_1 \qquad (3.9)$$

where P_1 is Paasche's index formula. By combining (3.6) and (3.9) one gets

$$P_o > I > P_1$$

This conclusion is based on the assumptions that (a) Q_o and Q_2 are situated on the same indifference curve and that (b) the indifference curve is convex toward the origin. Since the actual index lies between the two indexes (P_o and P_1), a geometric

average of the two may be taken as the index's value.

IV. THE COMPUTATION OF A CONSUMER PRICE INDEX

The United States Bureau of Labor Statistics computes a Consumer Price Index which indicates the percentage change in the price of a bill of commodities typically purchased by wage earners and clerical workers in forty-six cities. Purchases are divided into major groups, such as, (a) Food, (b) Housing, (c) Apparel, (d) Transportation, (e) Medical Care, (f) Personal Care, (g) Reading and Recreation and (h) Other goods and services.

Procedures and time of sampling differ among the groups. For example, food items are sampled in large cities on the 15th of each month in the local stores, but they are sampled less frequently in small cities. In the case of housing, information is obtained from tenants by mail questionaire, from builders and owners by personal interview, and from the Department of Commerce. A survey on house furnishing is done quarterly and a survey on rent is done semi-annually.

A separate index is first computed for each group and the group indices are later combined into a composite index. The relative weight of each group in the composite index is proportional to the budget obtained from the Consumer Expenditure Survey. Within a major group there are two or more subgroups of commodities. Each of the subgroups may also contain a number of composite items. A typical formula for computing the index of a group, say food, is given in the Review of the National Bureau of Economic Research, Inc., 1961, and is reproduced here: Let the number of cities sampled be n. The population weight

assigned to the i^{th} city is w_i, $i = 1,2, \ldots n$; $\sum w_i = 1$. Within the i^{th} city, let the base weight of a specific food item be ${}_iw^{(o)}_{jklm}$, where (o) represents the base time period and the subscripts j, k, l, m, represent successively the subdivisions of the item. Quantities ${}_iw^{(o)}_{jklm}$ are relative weights so that

$$\sum_{j,k,l} {}_iw^{(o)}_{jklm} = 1$$

For an item m which is representative of subsub group jkl, say jklm in city i, at time t, a number of price quotations are obtained, for example, ${}_ip^{(t)}_{jklx_1}$, ${}_ip^{(t)}_{jklx_2}$, $\ldots$, ${}_ip^{(t)}_{jklx_s}$ with an average ${}_i\bar{p}^{(t)}_{jklx}$. The price relative for this "specified-in-detail" item is

$$ {}_iR^{(t)}_{jklx} = \frac{{}_i\bar{p}^{(t)}_{jklx}}{{}_i\bar{p}^{(o)}_{jklx}} \tag{4.1}$$

where

$$ {}_iR^{(t)}_{jklx} \text{ approximates } {}_iR^{(t)}_{jkl} = \frac{\sum_m {}_iw^{(o)}_{jklm} \; {}_i\bar{p}^{(t)}_{jklm}}{{}_iw^{(o)}_{jkl}} \tag{4.2}$$

and summation is over all "specified-in-detail" items in the jkl^{th} subsub group. The city's index for the jk^{th} sub-group is

$$ {}_iR^{(t)}_{jk} = \frac{\sum_l {}_iw^{(o)}_{jkl} \dfrac{{}_i\bar{p}^{(t)}_{jklx}}{{}_i\bar{p}^{(o)}_{jklx}}}{{}_iw^{(o)}_{jk}} \tag{4.3}$$

and

$$ {}_iR^{(t)} = \sum_{i,j,k} {}_iw^{(o)}_{jkl} \frac{{}_i\bar{p}^{(t)}_{jklx}}{{}_i\bar{p}^{(o)}_{jklx}} \qquad (4.4) $$

No division is necessary for the all-item index since ${}_iw^{(o)}_{jkl} = 1$ when all major groups, sub-groups etc., are considered.

The Consumer Expenditure Survey data are used to determine the average dollar expenditure of an item jklm at time (o). Suppose this is called ${}_i\bar{v}_{jkl}$. Then the hypothetical base quantity associated with jklx item that will account for expenditure group jkl is

$$ {}_i\bar{q}^{(o)}_{jklx} = \frac{{}_i\bar{v}^{(o)}_{jkl}}{{}_i\bar{p}^{(o)}_{jklx}} \qquad (4.5) $$

$$ {}_iR^{(t)} = \frac{\sum_{j,k,l} {}_i\bar{q}^{(o)}_{jklx} \; {}_i\bar{p}^{(t)}_{jklx}}{\sum_{j,k,l} {}_i\bar{q}^{(o)}_{jklx} \; {}_i\bar{p}^{(o)}_{jklx}} \qquad (4.6) $$

and

$$ R^{(t)} = \frac{\sum_i W_i \sum_{j,k,l} {}_i\bar{q}^{(o)}_{jklx} \; {}_i\bar{p}^{(t)}_{jklx}}{\sum W_i \sum_{j,k,l} {}_i\bar{q}^{(o)}_{jklx} \; {}_i\bar{p}^{(o)}_{jklx}} \qquad (4.7) $$

where W_i is the actual population figure in the i^{th} city.

The following table is presented for a comparison among the values obtained when the six formulae given earlier are used to

compute an index of price change. Some expenditures which had been taken from a "Family Budget of City Workers" and which were recorded in the monthly Labor Review of February 1951, Vol. 72, No. 2, pp. 152-155, are used here as the relative weights of the prices. Further, the relevant relative prices have been copied from the Economic Report of the President, of July 1948 and January 1950. These relative prices applied to the whole nation but the budget applied to Kansas City area only. Therefore, none of the indices can be taken as the correct index; each is cited as an illustration only. The two months of comparison are June, 1947 and October, 1949, where the former is taken as the base, i.e. period-o.

Table 1. Index of Sales of Goods and Services for June 1947 and October 1949. (base: 1935-1939)

	Food	Clothing	Rent	Fuel, Electricity, Household Operation	House Furnishings	Other Goods & Services, Medical Care, Transportation Misc.
June '47	329.0	300.3	177.0	135.0	444.2	253.1
Oct. '49	320.5	283.7	140.7	141.0	248.7	272.5

Laspeyres' Aggregative Index: $$L_{o1} = \frac{\sum q_{io}p_{i1}}{\sum q_{io}p_{io}} = 1.0513$$

Paasche's Aggregative Index: $$P_{o1} = \frac{\sum q_{i1}p_{i1}}{\sum q_{i1}p_{io}} = 1.0578$$

Table 2. Index of Consumer Prices for June 1947 and October 1949. (base: 1935-1939)

	Food	Clothing	Rent	Fuel, Electricity Household Operation	House Furnishings	Other Goods & Services, Medical Care, Transportation Misc.
June '47	190.5	185.7	109.2	117.7	182.6	139.1
Oct. '49	200.6	186.8	121.5	138.4	185.2	155.2

Cross of Laspeyres' and Paasche's

Arithmetic Cross $\frac{1}{2}(L_{01} + P_{01}) = 1.0546$

Geometric Cross $\sqrt{L_{01} \cdot P_{01}} = 1.0545$

Cross-weight Aggregative Index:

Arithmetic $$\frac{\sum (q_{io} + q_{i1})p_{i1}}{\sum (q_{io} + q_{i1})p_{io}} = 1.0686$$

Geometric $$\frac{\sqrt{\sum q_{io}q_{i1}}\ p_{i1}}{\sqrt{\sum q_{io}q_{i1}}\ p_{io}} = 1.0547$$

V. DISCUSSION

Comparison between the Cost of Living Index and Consumer Price Index

It has been shown that a Consumer Price Index (see part III) can be used to approximate a True Cost of Living Index only in a special situation. The assumptions for the computation of a Cost of Living Index are: (a) A group of individuals with similar tastes exists, (b) these individuals receive equal real income during the two periods of comparison, and (c) the state of technology is given and remains constant during the periods.

Every assumption will not be valid in a dynamic economic system where people's tastes change with improved technology and the standard of living is continually rising. The Cost of Living Index indicates the change in an average individual's expenditures on a given collection of commodities whereas the Consumer Price Index indicates the change in the prices of commodities typically bought by a group of people. No assumption is made about the consumption. However, the bill of commodities used in the computation is often revised to keep step with the changes in purchases of the group. Thus the Consumer Price Index reflects the change in the tastes of the group. The True Cost of Living Index is suited for a theoretical analysis where the operation of one factor is observed while the other factors are assumed dormant. The Consumer Price Index refers to the operation of one factor-price in a dynamic economy system, without making any assumptions about any other factor.

The Relation of a Price Index to a Quantity Index

Most studies of index numbers are made with prices because of the relative economy in collecting an adequate sample and the wider application of index numbers to prices. Further, the characteristics of a good quantity index are different from those of a good price index. To compute a quantity index, one has to obtain as complete a coverage as possible, for, the movements in the quantities of different commodities may differ, and the movements in production of a given commodity by the various firms, may differ too. But, in the case of a price index, it is not so important to get a complete coverage, for prices, because close substitutes charged by the various firms are likely to show similar movements, and it is possible to apply a sampling procedure to the construction of price indices for varieties of a product or related products.

If it is desired to find the quantity and price index numbers, as it is the case in the National Account, and the value series are available for the groups of net outputs and final expenditures, then one can compute a current-weighted or base-weighted price index and divide it into the total value index to get the base-weighted or the current-weighted quantity index, respectively (Stone, 1956).

ACKNOWLEDGEMENTS

I would like to thank Dr. Liang-Shing Fan, of the Economics Department, for his suggestions at the initial time on the organization of the material of this report.

I am grateful to Dr. Leslie F. Marcus for his encouragement and guidance.

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INDEX NUMBERS

by

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B. S., Kansas State University, 1963

AN ABSTRACT OF A MASTER'S REPORT

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Statistics

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1966

An index number is a special type of ratio which is used to indicate an average change in a group of related variables. The primary purpose for determining changes in variables, such as prices and quantities of consumers' goods, is to estimate changes in the welfare of the people of a country. Guided by such indices, the government, or groups of concerned people, may take corrective measures to ensure a continual increase in the welfare of the people and to ensure a more even distribution of wealth. For example, the United States Department of Agriculture computes an index number to measure changes in the average prices of farm products and another index to measure changes in the average prices of non-farm products. The ratio of the first index to the second is a measure of the change in purchasing power of farm products and as a basis from which the government's price support is determined. In the analysis of cyclical change in the physical volume of production, a series of indices are used by economists in order to determine the causes of disruptive variations in business cycles.

Index numbers may be applied to prices, production, volume of trade or any other factor subject to temporal or spatial variation. A clear statement of the purpose for computing the individual index number is necessary to determine the appropriate index construction procedure. The problems of obtaining a representative sample, the choice of weights, the choice of the periods to be compared, and the method of averaging the variables need to be carefully considered. Many formulae have been sug-

gested for the computation of index numbers. Irving Fisher tested 134 such formulae for adequacy according to desirable properties and eliminated 121 by two tests alone. Through time only Laspeyres' formula, Paasche's formula and the Arithmetic cross-weight aggregative formula have survived. Laspeyres' formula is a favorite because it is easy to interpret and requires the least amount of data for its computation.

Economists have turned to the comparison of utility because an adequate formula has not been obtained for computing change in price level, and for estimating changes in real income, which are related to changes in the level of welfare of the people. Through the use of utility concepts, it has been shown that the 'True Cost of Living Index', which is a ratio of money incomes which yield the same satisfaction in two situations can be approximated by the geometric mean of Laspeyres' and Paasche's price indices. It is assumed that all economic variables other than prices are held constant. Variables such as taste, and technology also affect consumption, but they are difficult to include in an analysis and limit its applicability.

The Consumer Price Index which was called a Cost of Living Index until 1945, has been adopted for practical application. This index is sometimes mistaken for the 'True Cost of Living Index', therefore, the difference between the two index numbers needs to be emphasized. The assumptions underlying the computation of the True Cost of Living Index are: (1) constant tastes; (2) constant state of technology; and (3) equivalent incomes for

the two periods of comparison. The Bureau of Labor Statistics, which computes the Consumer Price Index, does not make such assumptions. Their Consumer Price Index indicates only the changes in prices of commodities typically purchased by urban wage earners and clerical workers. The technology continually changes and the real incomes of the groups change also. The Bureau revises the items included in the group of commodities sampled for the computation of the index as people vary their purchases, which may partially account for changes in taste. The Consumer Price Index is used to deflate the Index of Relative Earnings to obtain an Index of Real Wages which measures change in the well-being of wage earners.